

ANNEX 8

In-service analysis of structural sections and elements subject to simple bending

1 Scope

This Annex sets out the expressions allowing the various parameters governing the sectional behaviour of rectangular (box) and T-sections, under linear cracking conditions, to be determined: depth of the neutral fibre X , stress state of the reinforcement fibres σ_{s1} and σ_{s2} and of the concrete σ_c , strains in the reinforcements ε_{s1} and ε_{s2} , and stiffness values.

The expressions in this Annex allow the stresses in the tensioned reinforcement (σ_s , σ_{st}) to be determined in order to check the Cracking Limit State (Article 49) and they also allow the cracked inertia (I_f) to be determined in order to check the Deformation Limit State (Article 50).

This Annex also deals with checking the Serviceability Limit States (cracking and deformations) in reinforced or prestressed linear elements, composed of one or more concretes, in which the construction stages must be borne in mind. Some of the expressions contained in this Annex are generalisations of those in the main articles, for example the expression relating to equivalent inertia which is a generalisation of the Branson formula for the case of compound and/or prestressed members.

Finally, expressions are given for calculating delayed deflection. These are more appropriate for high-strength concretes than those in the main articles and are useful in cases where the determination of the deflection needs to be refined.

2 Calculation of sections in service with cracking.

2.1 Basic assumptions

The assumptions made in order to produce the expressions given are as follows:

- The plane of strain remains plane after deformation.
- Perfect bond between concrete and steel.
- Linear behaviour for the compressed concrete.

$$\sigma_c = E_c \varepsilon_c$$

- The tensile strength of the concrete is ignored.
- Linear behaviour for the steels, under both tension and compression.

$$\sigma_{s1} = E_s \varepsilon_{s1}$$

$$\sigma_{s2} = E_s \varepsilon_{s2}$$

2.2 Rectangular section

For rectangular sections, the values of the parameters defining the sectional behaviour (Figure A.8.1) are:

- Relative depth of the neutral fibre

$$\frac{X}{d} = n \rho_1 \left(1 + \frac{\rho_2}{\rho_1} \right) \left(-1 + \sqrt{1 + \frac{2 \left(1 + \frac{\rho_2 d'}{\rho_1 d} \right)}{n \rho_1 \left(1 + \frac{\rho_2}{\rho_1} \right)^2}} \right)$$

$$\text{if } \rho_2 = 0 \Rightarrow \frac{X}{d} = n \rho_1 \left(-1 + \sqrt{1 + \frac{2}{n \rho_1}} \right)$$

- Cracked inertia

$$I_f = n A_{s1} (d - X) \left(d - \frac{X}{3} \right) + n A_{s2} (X - d') \left(\frac{X}{3} - d' \right)$$

where:

$$n = \frac{E_s}{E_c}$$

$$\rho_1 = \frac{A_{s1}}{bd}$$

$$\rho_2 = \frac{A_{s2}}{bd}$$

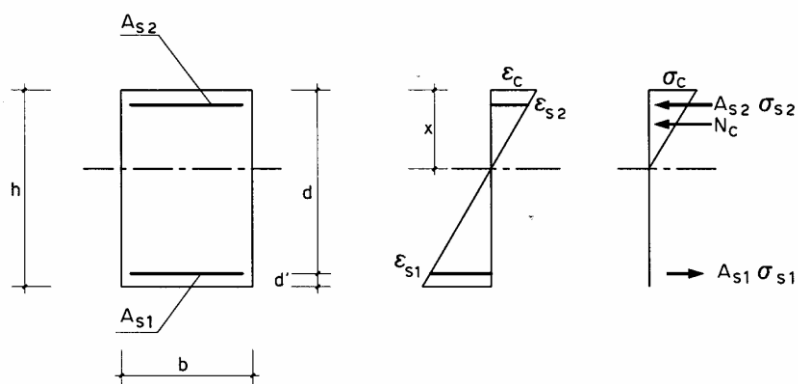


Figure A.8.1

2.3 T-section

For T-sections, the values of the parameters defining the sectional behaviour (Figure A.8.2) can be determined using the expressions defined below.

$$\delta = \frac{h_0}{d}$$

$$\xi = \delta \left(\frac{b}{b_0} - 1 \right)$$

$$\rho_1 = \frac{A_{s1}}{b d}$$

$$\rho_2 = \frac{A_{s2}}{b d}$$

$$\beta = \xi + n(\rho_1 + \rho_2) \frac{b}{b_0}$$

$$\alpha = 2 n(\rho_1 + \rho_2 \frac{d'}{d}) \frac{b}{b_0} + \xi \delta$$

$$1^\circ. \quad n \rho_1 \leq \frac{1}{2} \frac{\delta^2 + 2n\rho_2(\delta - d' / d)}{(1 - \delta)}$$

The values of X/d and l_f will be determined using the expressions in section 3 for rectangular sections, taking the width of the compression flange as the section width.

$$2^\circ. \quad n \rho_1 > \frac{1}{2} \frac{\delta^2 + 2n\rho_2(\delta - d' / d)}{(1 - \delta)}$$

- Relative depth of the neutral fibre

$$\frac{X}{d} = \beta \left(-1 + \sqrt{1 + \frac{\alpha}{\beta^2}} \right)$$

- Cracked section inertia

$$I_f = I_c + n A_{s1} (d - X)^2 + n A_{s2} (X - d')^2$$

$$I_c = b h_0 \left[\frac{h_0^2}{12} + \left(X - \frac{h_0}{2} \right)^2 \right] + \frac{b_0 (X - h_0)^3}{3}$$

In case 1, the position of the neutral fibre in the cracked section is included within the compression flange and, as a result, the expressions for calculating the parameters governing the sectional behaviour are those corresponding to the rectangular section

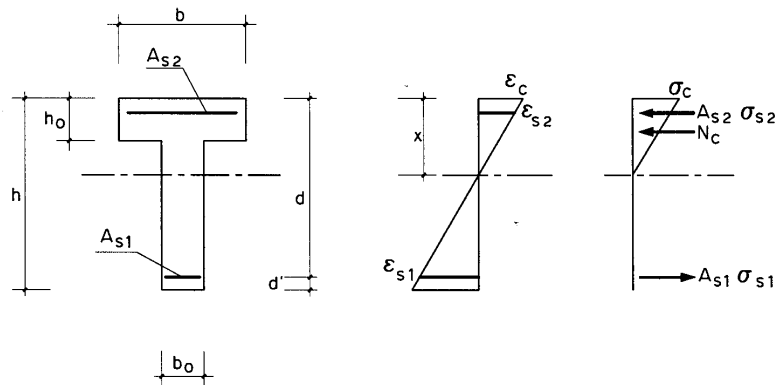


Figure A.8.2.

2.4 Curvature and stresses

The curvature and stresses in the concrete and in the various steel fibres can be determined using the following expressions:

- Curvature

$$\frac{1}{r} = \frac{M}{E_c I_f}$$

- Compressive stress in the most compressed concrete fibre

$$\sigma_c = \frac{M X}{I_f}$$

- Stress in the reinforcements

$$\sigma_{s1} = n \sigma_c \frac{d - X}{X}$$

$$\sigma_{s2} = -n \sigma_c \frac{X - d'}{X}$$

3. Checking the cracking in one-way floor slabs composed of precast elements and site-cast concrete.

For structures composed of precast elements and site-cast concrete, the stress calculation must take account of the various stages through which these structural elements pass in terms of both the acting loads and also the support and load bearing conditions. The following must therefore be taken into account:

- the self-weight of the precast element (prestressed hollow-core slab or beam, if prestressed) calculated as a simple supported element without intermediate shores, acting on the simple section;
- the self-weight of the rest of the floor slab will act on a continuous beam with as many spans as there are intermediate shores plus one, acting on the simple section;
- the effect of unshoring (application of the reactions due to the intermediate shores on the final configuration), acting on the compound section;
- application of the permanent load and overload, acting on the final configuration and compound section.

In particular, the self-weight of the prestressed elements (beams or hollow-core slabs) must not be assumed to be continuous or supported. Rather, the isostatic bending moment corresponding to their on-site location between end supports, without any intermediate shores and acting on the isolated element (simple section), must be taken into account.

If the joist is reinforced, its self-weight is not regarded as an independent stage but is included, however, in the next stage, such as the rest of the self-weight of the floor slab.

The above process can make determining the stresses more complex. In the absence of other criteria, the procedure indicated below can be followed to simplify matters.

The stresses can be determined using Navier's Hypothesis and the following sections: simple, compound uncracked and cracked corresponding to each situation. For sections subject to positive moments, the checking moment will be given by:

$$M_p = (g_1 + (1 - K_1)g_2) \frac{L^2}{8} + (g_3 + q) \frac{L_0^2}{8}$$

and for negative moments:

$$M_n = [K_2 g_2 + g_3 + q] \frac{L_0^2}{8}$$

where:

- α Ratio between section modulus (W_{1h}' / W_{1h})
- W_{1h} Section modulus of the simple section. Figure A.8.3
- W_{1h}' Section modulus of the compound section. Figure A.8.3
- K_1, K_2 Factors, according to Table A.8.3
- L Span of the floor slab
- L_0 Distance between points of zero moment, corresponding to the continuous situation of the floor slab
- g_1 Variable corresponding to the self-weight of the precast element, if prestressed, and which will take a zero value in the case of reinforced elements
- g_2 Variable corresponding to the self-weight of the beam if reinforced, to the self-weight of the site-cast concrete and, where applicable, to the infill blocks
- g_3 Variable corresponding to the permanent load (for example, flooring)
- q Overloads

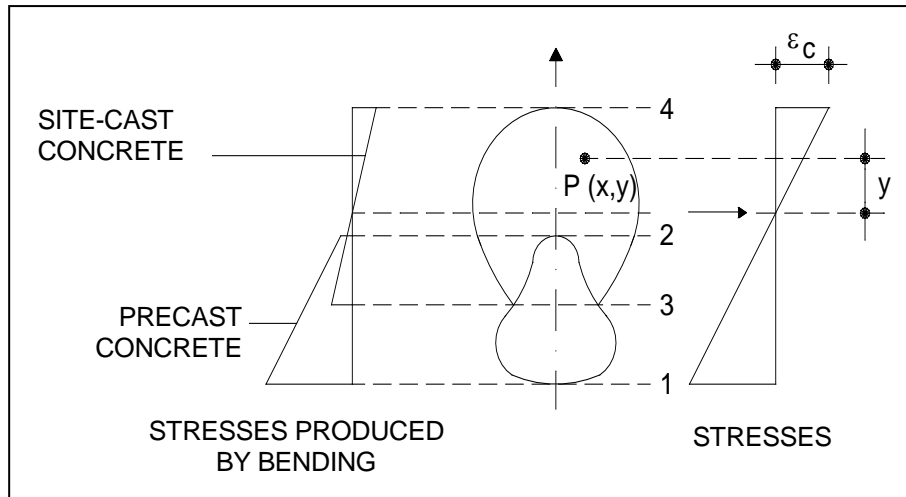


Figure A.8.3.
Cracking Limit States

Table A.8.3

Case		K_1	K_2
	Without secondary supports	0	0
	One row of secondary supports	$1,25 \left[1 - \frac{5}{16} \frac{(\alpha - 1)g_2}{\alpha(g_1 + g_2) + g_3 + q} \right]$	1,25
	Two rows of secondary supports at thirds along the span	0,98	0,98
	Two rows of secondary supports at 0,4 L from each support	1,06	1,06
	Three or more rows of secondary supports	1	1

Attention is drawn to the importance of carrying out a correct floor slab shoring procedure, without which the above formulas will not be valid. Therefore, in the case of reinforced elements, levelled shores must be placed all at the same height. On the other hand, in the case of floor slabs with prestressed elements, the shores are placed against the lower edge of the precast element after this has been put into place, supported at its ends.

4. Simplified calculation of instantaneous deflections in prestressed members or those constructed in stages

The Branson formula given in Article 50.2.2 for calculating the instantaneous deflection in the case of reinforced concrete beams constructed in a single stage can be generalised for the case of reinforced or prestressed members constructed in one or more stages or composed of precast elements and site-cast concrete, such as one-way floor slabs. The equivalent inertia of the section in question can be determined using the expression:

$$I_e = \left(\frac{M_f - M_0}{M_a - M_0} \right)^3 I_b + \left(1 - \left(\frac{M_f - M_0}{M_a - M_0} \right)^3 \right) I_f \leq I_b$$

where:

- I_b Moment of inertia of the gross section.
- I_f Moment of inertia of the cracked section under simple bending; this is determined by ignoring the tension zone of the concrete and standardising the areas of the active and passive reinforcements by multiplying these by the coefficient of equivalence.
- M_a Maximum bending moment applied to the section up to the instant when the deflection is assessed.
- M_f Cracking moment, calculated as follows:

$$M_f = W (f_{ct,f} + \sigma_{cp}) + M_v \left(1 - \frac{W}{W_v} \right)$$

where:

- W Section modulus about the most highly tensioned fibre of the section, which will be:
 - that of the precast member (W_v), in the case of unshored construction, where the deflection is calculated under the self-weight of this member or the site-cast concrete.
 - that of the floor slab (W_f) at any stage of shored construction and in service.
- $f_{ct,m,fl}$ Mean flexural strength of the concrete defined in Article 39.1.
- σ_{cp} Prior stress in the lower fibre of the precast member, produced by the prestressing.
- M_v Moment due to the loads acting on the precast member before working together with the site-cast concrete, for which the value is:
 - For unshored construction, the moment due to the self-weight of the precast member and to the weight of the site-cast concrete.
 - For unshored construction, zero if the member is reinforced and the moment due to its self-weight if this is prestressed.
 - Zero in the end sections subject to negative moments.
- M_0 Bending moment associated with the zero curvature situation of the section, with a value of:

$$M_0 = P \cdot e \cdot \beta - M_v \cdot (\beta - 1)$$

where:

- P Absolute value of the prestressing force, where this exists, which may be taken as equal to 90% of the initial prestressing force.

- e Eccentricity of the equivalent prestressing tendon, in the section under study, with an absolute value, about the centre of gravity of the beam or hollow-core slab. β Ratio between the gross inertia of the floor slab section in the construction stage in which the deflection is calculated and the gross inertia of the section of the precast member, greater than or equal to one. In unshored construction, where the deflection is calculated under the self-weight of the precast member or site-cast concrete, $\beta=1$.

The value of the cracked inertia given in the formula is the lowest which the section under study may historically have reached during the construction process, including due to the application of loads which are subsequently removed, as is the case with the shoring of upper slab floors on a lower unshored floor.

The bending force M_0 is designed to take account of the prestressing effect and the evolution of the section in the calculation of the equivalent stiffness in the cracked stage, in order to start with zero curvature. It may be observed that, when there is only a precast member, without any site-cast concrete, $\beta = 1$ and $M_0 = P \cdot e$.

In the span centre section of floor slabs with prestressed hollow-core slabs or beams, the following approximate expression may be used to calculate the cracked inertia I_f which takes account of the reduction in stiffness as the stresses increases:

$$I_f = I_{f0} + \alpha \cdot (I_b - I_{f0}) \leq I_b$$

where:

I_{f0} Inertia of the cracked section under simple bending, calculated taking into account the active reinforcement as if this were passive, i.e. taking into account a zero prestressing force.

I_b Inertia of the gross concrete section of the floor slab section.

α Inertia interpolation factor whose value, always between 0 and 1, is:

$$\alpha = \frac{\sigma_{cp}}{\frac{M_v}{W_v} + \frac{M_a - M_v}{W_f} - f_{ct,f}}$$

W_v , σ_{cp} , M_v and M_a have the same meaning as indicated above.

In the case of floor slabs with reinforced concrete precast members, the inertia of the cracked section is $I_f = I_{f0}$, given that $\alpha = 0$.